

# New physics contributions in $B \rightarrow \pi \tau \bar{\nu}$ and $B \rightarrow \tau \bar{\nu}$

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We study possible new physics contributions in  $B \rightarrow \pi \tau \bar{\nu}$  and  $B \rightarrow \tau \bar{\nu}$  employing the model-independent effective Lagrangian that describes the quark-level transition  $b \rightarrow u \tau \bar{\nu}$  at low energies. The decay rate of  $B \rightarrow \pi \tau \bar{\nu}$  and its theoretical uncertainty are evaluated using the  $B \rightarrow \pi$  form factors given by recent lattice QCD studies. Comparing theoretical results with the current experimental data,  $\mathcal{B}(B \rightarrow \pi \tau \bar{\nu}) < 2.5 \times 10^{-4}$  and  $\mathcal{B}(B \rightarrow \tau \bar{\nu}) = (1.14 \pm 0.22) \times 10^{-4}$ , we obtain constraints on the Wilson coefficients that quantify potential new physics. We also present the expected sensitivity of the SuperKEKB/Belle II experiment.  
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## 1. Introduction

Discrepancy of  $\sim 4\sigma$  between experimental results and the standard model (SM) exists in the semitauonic  $B$  meson decays,  $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}_\tau$  [1–5]. This anomaly is interesting apart from its statistical significance in the sense that it suggests a manifestation of new physics beyond the SM in the tree-level charged current SM processes involving the third-generation quark and lepton.

Since the interaction of quarks and leptons in the third generation might be a clue to new physics, it is natural to search for a similar effect in the  $b \rightarrow u \tau \bar{\nu}$  transition.<sup>1</sup> The evidence of the purely tauonic decay,  $B^- \rightarrow \tau^- \bar{\nu}$ , has been found by both the BaBar and Belle collaborations and the combined value of their results of the branching fraction is  $\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}) = (1.14 \pm 0.22) \times 10^{-4}$  [6], which is consistent with the SM prediction. Recently, the Belle collaboration reported on the semitauonic decay,  $\bar{B}^0 \rightarrow \pi^+ \tau^- \bar{\nu}$  [7]. They observed no significant signal and obtained an upper limit of the branching fraction as  $\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \tau^- \bar{\nu}) < 2.5 \times 10^{-4}$  at the 90% confidence level (CL). As given in Ref. [7], the observed signal strength is  $\mu = 1.52 \pm 0.72$ , where  $\mu = 1$  corresponds to the branching fraction in units of  $10^{-4}$ , and thus one obtains

$$\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \tau^- \bar{\nu}) = (1.52 \pm 0.72 \pm 0.13) \times 10^{-4}, \quad (1)$$

where the second error comes from the systematic uncertainty (8%). Since the SM predicts  $\sim 0.7 \times 10^{-4}$ , a new physics contribution of similar magnitude to the SM is allowed. We expect that the SuperKEKB/Belle II experiment will provide important information on possible new physics in  $\bar{B}^0 \rightarrow \pi^+ \tau^- \bar{\nu}$  as well as  $B^- \rightarrow \tau^- \bar{\nu}$ .

<sup>1</sup> The charge-conjugated mode is implicit in the present work.

Sensitivity to new physics effects depends on the precision of theoretical predictions as well as experimental errors. The major uncertainty in the SM prediction of  $\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \tau^- \bar{\nu})$  is ascribed to the Cabibbo–Kobayashi–Maskawa matrix element  $|V_{ub}|$  and the  $B \rightarrow \pi$  hadronic form factors. In order to reduce these uncertainties, it is useful to introduce the ratio of branching fractions [8–10],

$$R_\pi = \frac{\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \tau^- \bar{\nu})}{\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \ell^- \bar{\nu})}, \quad (2)$$

as in the study of  $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}$ . Although  $|V_{ub}|$  cancels out in this ratio, there remains the uncertainty due to the form factors. Using the result of the recent lattice QCD study [11], in which the relevant form factors are obtained by fitting both the lattice amplitude and the experimental data of  $B \rightarrow \pi \ell \bar{\nu}$  [12–15], the SM prediction is obtained as  $R_\pi^{\text{SM}} = 0.641 \pm 0.016$  [10,16].<sup>2</sup> The experimental value is estimated as  $R_\pi^{\text{exp}} \simeq 1.05 \pm 0.51$ , where  $\mathcal{B}(B \rightarrow \pi \ell \bar{\nu}) = (1.45 \pm 0.02 \pm 0.04) \times 10^{-4}$  [6] is used.<sup>3</sup> New physics effects in  $R_\pi$  and related quantities are studied in the literature. The effect of the charged Higgs boson, which appears in the supersymmetric extension of the SM, is studied in Refs. [8–10]. The supersymmetric SM without  $R$  parity is also studied in  $b \rightarrow u$  (semi)leptonic processes [18].

In the present work, we study new physics effects in  $B \rightarrow \pi \tau \bar{\nu}$  and  $B \rightarrow \tau \bar{\nu}$  using the model-independent effective Lagrangian that describes the  $b \rightarrow u \tau \bar{\nu}$  transition at low energies. Comparing with the current experimental data, we obtain constraints on the Wilson coefficients that quantify potential new physics. The theoretical uncertainties of  $R_\pi$  in both the SM and new physics contributions are examined with the lattice QCD results. We also discuss the prospects of the new physics search in  $B \rightarrow \pi \tau \bar{\nu}$  and  $B \rightarrow \tau \bar{\nu}$  at SuperKEKB/Belle II.

This paper is organized as follows. In Sect. 2, we will introduce the  $b \rightarrow u \tau \bar{\nu}$  effective Lagrangian that describes possible new physics contributions to  $B \rightarrow (\pi) \tau \bar{\nu}$ . We will also provide the relevant rate formulae and theoretical uncertainties derived from errors of form factor parameters given by lattice studies. In Sect. 3, we will present current constraints on new physics from  $B \rightarrow \pi \tau \bar{\nu}$  and  $B \rightarrow \tau \bar{\nu}$ , and discuss future prospects at SuperKEKB/Belle II. A summary will be given in Sect. 4.

## 2. Formulae of new physics effects

### 2.1. Effective Lagrangian

In order to represent possible new physics effects at low energies, we adopt the model-independent approach with use of an effective Lagrangian [19,20]. As in our previous work [19], we assume that  $b \rightarrow u \tau \bar{\nu}$  is affected by new physics while  $b \rightarrow u \ell \bar{\nu}$  ( $\ell = e, \mu$ ) is practically described by the SM. The effective Lagrangian used in this work is given by

$$-\mathcal{L}_{\text{eff}} = 2\sqrt{2}G_F V_{ub} \left[ (1 + C_{V_1}) \mathcal{O}_{V_1} + C_{V_2} \mathcal{O}_{V_2} + C_{S_1} \mathcal{O}_{S_1} + C_{S_2} \mathcal{O}_{S_2} + C_T \mathcal{O}_T \right], \quad (3)$$

<sup>2</sup> Reference [17] gives a different SM prediction. Our evaluation below agrees with Refs. [10,16].

<sup>3</sup> This is not the same way to obtain the experimental result of  $R_{D^{(*)}} = \mathcal{B}(\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}) / \mathcal{B}(\bar{B} \rightarrow D^{(*)} \ell \bar{\nu})$  [1–5]. The ratios  $R_{D^{(*)}}$  are directly extracted with the signal events in the numerator and the normalization ones in the denominator both involved in the same event sample.

where the four-fermion operators are defined as

$$\mathcal{O}_{V_1} = (\bar{u}\gamma^\mu P_L b)(\bar{\tau}\gamma_\mu P_L \nu_\tau), \quad (4)$$

$$\mathcal{O}_{V_2} = (\bar{u}\gamma^\mu P_R b)(\bar{\tau}\gamma_\mu P_L \nu_\tau), \quad (5)$$

$$\mathcal{O}_{S_1} = (\bar{u}P_R b)(\bar{\tau}P_L \nu_\tau), \quad (6)$$

$$\mathcal{O}_{S_2} = (\bar{u}P_L b)(\bar{\tau}P_L \nu_\tau), \quad (7)$$

$$\mathcal{O}_T = (\bar{u}\sigma^{\mu\nu} P_L b)(\bar{\tau}\sigma_{\mu\nu} P_L \nu_\tau), \quad (8)$$

and  $C_X$  ( $X = V_{1,2}, S_{1,2}, T$ ) denotes the Wilson coefficient of  $\mathcal{O}_X$  normalized by  $2\sqrt{2}G_F V_{ub}$ . We consider only  $\tau$ - $\nu_\tau$  currents for simplicity though the neutrino flavor could be the first or second generation in some new physics models. One may translate the following result of  $C_X$  for  $\nu_{\ell=\tau}$  into that for  $\nu_{\ell \neq \tau}$  by replacing  $C_X \rightarrow i|C_X|$ . Since  $(\bar{u}\sigma^{\mu\nu} P_R b)(\bar{\tau}\sigma_{\mu\nu} P_L \nu_\ell) = 0$ , there is only one possible tensor operator unless right-handed neutrinos are included in the low-energy particle spectrum. The SM contribution is represented by the unit coefficient of  $\mathcal{O}_{V_1}$ , namely putting  $C_X = 0$  for all  $X$ 's gives the SM.

In this paper, we focus on new physics effects in  $B \rightarrow \pi \tau \bar{\nu}_\tau$  and  $B \rightarrow \tau \bar{\nu}_\tau$ . Other processes such as  $B \rightarrow V \tau \bar{\nu}_\tau$  for  $V = \rho, \omega$  might become useful in future, but for now no experimental data are available.

## 2.2. $\bar{B}^0 \rightarrow \pi^+ \tau^- \bar{\nu}_\tau$

The  $B \rightarrow \pi$  transition caused by the effective Lagrangian in Eq. (3) is described by the hadronic matrix elements of the quark currents involved in the four-fermion operators:

$$\langle \pi(p_\pi) | \bar{u}\gamma^\mu b | \bar{B}(p_B) \rangle = f_+(q^2) \left[ (p_B + p_\pi)^\mu - \frac{m_B^2 - m_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu, \quad (9)$$

$$\langle \pi(p_\pi) | \bar{u}b | \bar{B}(p_B) \rangle = (m_B + m_\pi) f_S(q^2), \quad (10)$$

$$\langle \pi(p_\pi) | \bar{u} i \sigma^{\mu\nu} b | \bar{B}(p_B) \rangle = \frac{2}{m_B + m_\pi} f_T(q^2) [p_B^\mu p_\pi^\nu - p_B^\nu p_\pi^\mu], \quad (11)$$

where  $q^\mu = (p_B - p_\pi)^\mu = (p_\tau + p_\nu)^\mu$ , and  $f_{+,0,S,T}(q^2)$  are form factors. We note that the axial-vector (pseudoscalar) part of  $V_{1,2}$  ( $S_{1,2}$ ),  $\bar{u}\gamma^\mu \gamma^5 b$  ( $\bar{u}\gamma^5 b$ ), does not contribute to the transition, and  $\langle \pi(p_\pi) | \bar{u}\sigma^{\mu\nu} \gamma^5 b | \bar{B}(p_B) \rangle$  is expressed by  $f_T(q^2)$  with  $\sigma^{\mu\nu} \gamma^5 = -\frac{i}{2} \epsilon^{\mu\nu\alpha\beta} \sigma_{\alpha\beta}$ .<sup>4</sup> We employ the vector and tensor form factors  $f_{+,0,T}$  given by recent lattice QCD studies [11,21]. As for the scalar form factor  $f_S$ , since no lattice evaluation is available at present, we utilize the quark equation of motion to relate  $f_S$  to  $f_0$ , namely  $f_S(q^2) = f_0(q^2)(m_B - m_\pi)/(m_b - m_u)$ .

The differential branching fractions of  $B \rightarrow \pi \tau \bar{\nu}_\tau$  for given  $\tau$  helicities, defined in the rest frame of the lepton pair, are written as

$$\frac{d\mathcal{B}_\tau^-}{dq^2} = N_B \left| (1 + C_{V_1} + C_{V_2}) \sqrt{q^2} H_{V_+} + 4C_T m_\tau H_T \right|^2, \quad (12)$$

<sup>4</sup> We take  $\epsilon^{0123} = -1$ .

for  $\lambda_\tau = -1/2$ , and

$$\begin{aligned} \frac{d\mathcal{B}_\tau^+}{dq^2} = \frac{N_B}{2} & \left[ \left| (1 + C_{V_1} + C_{V_2})m_\tau H_{V_+} + 4C_T\sqrt{q^2}H_T \right|^2 \right. \\ & \left. + 3 \left| (1 + C_{V_1} + C_{V_2})m_\tau H_{V_0} + (C_{S_1} + C_{S_2})\sqrt{q^2}H_S \right|^2 \right], \end{aligned} \quad (13)$$

for  $\lambda_\tau = +1/2$ , with

$$N_B = \frac{\tau_{B^0} G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} (Q_+ Q_-)^{1/2} \left( 1 - \frac{m_\tau^2}{q^2} \right)^2, \quad (14)$$

where  $\tau_{B^0}$  is the neutral  $B$  meson lifetime and  $Q_\pm = (m_B \pm m_\pi)^2 - q^2$ . The hadronic amplitudes  $H$ 's are given by

$$H_{V_+} = \frac{(Q_+ Q_-)^{1/2}}{\sqrt{q^2}} f_+(q^2), \quad (15)$$

$$H_{V_0} = \frac{m_B^2 - m_\pi^2}{\sqrt{q^2}} f_0(q^2), \quad (16)$$

$$H_S = (m_B + m_\pi) f_S(q^2) = \frac{m_B^2 - m_\pi^2}{m_b - m_u} f_0(q^2), \quad (17)$$

$$H_T = \frac{(Q_+ Q_-)^{1/2}}{m_B + m_\pi} f_T(q^2), \quad (18)$$

where the bottom and up quark masses are taken as  $m_b = 4.2 \text{ GeV}$  and  $m_u = 0$  in the following numerical calculation. The differential branching fractions of  $B \rightarrow \pi \ell \bar{\nu}_\ell$  (for  $m_\ell = 0$ ) are obtained as

$$\frac{d\mathcal{B}_\ell^-}{dq^2} = \frac{d\mathcal{B}_\tau^-}{dq^2} \Big|_{m_\tau \rightarrow 0, C_X=0}, \quad (19)$$

$$\frac{d\mathcal{B}_\ell^+}{dq^2} = 0. \quad (20)$$

In the following, the ratio of the branching fractions,  $R_\pi$  in Eq. (2), is numerically calculated by

$$R_\pi = \frac{\int_{m_\tau^2}^{(m_B+m_\pi)^2} dq^2 \frac{d\mathcal{B}_\tau^+ + d\mathcal{B}_\tau^-}{dq^2}}{\int_0^{(m_B+m_\pi)^2} dq^2 \frac{d\mathcal{B}_\ell^-}{dq^2}}. \quad (21)$$

As mentioned above,  $|V_{ub}|$  cancels out in this ratio, but errors in the form factors cause the theoretical uncertainty in  $R_\pi$ .

The form factors  $f_+$ ,  $f_0$ , and  $f_T$  are parametrized with the use of the Bourrely–Caprini–Lellouch expansion as [11,21,22]

$$f_j(q^2) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{n=0}^{N_z-1} b_n^j \left[ z^n - (-1)^{n-N_z} \frac{n}{N_z} z^{N_z} \right], \quad (22)$$

$$f_0(q^2) = \sum_{n=0}^{N_z-1} b_n^0 z^n, \quad (23)$$

where  $j = +, T$ ,  $m_{B^*} = 5.325 \text{ GeV}$  is the  $B^*$  meson mass,  $b_n^{+,0,T}$  are expansion coefficients, and  $N_z = 4$  is the expansion order. The expansion parameter  $z$  is defined as

$$z \equiv z(q^2) = \frac{(t_+ - q^2)^{1/2} - (t_+ - t_0)^{1/2}}{(t_+ - q^2)^{1/2} + (t_+ - t_0)^{1/2}}, \quad (24)$$

where  $t_+ = (m_B + m_\pi)^2$  and  $t_0 = (m_B + m_\pi)(\sqrt{m_B} - \sqrt{m_\pi})^2$ . The combined fit to the experimental data of the  $q^2$  distribution of  $B \rightarrow \pi \ell \bar{\nu}_\ell$  and the lattice computation for the relevant amplitudes provides the “lattice+experiments” fitted values of  $b_n^{+,0,T}$  with errors and their correlations. According to Refs. [11,21], the result of the expansion coefficients  $\vec{b} = (b_0^+, b_1^+, b_2^+, b_3^+, b_0^0, b_1^0, b_2^0, b_3^0, b_0^T, b_1^T, b_2^T, b_3^T)^\top$  is summarized as

$$\vec{b}_{\text{lat.+exp.}} \equiv \vec{b}_0 \pm \delta \vec{b}, \quad (25)$$

where

$$\vec{b}_0 = (0.419, -0.495, -0.43, 0.22, 0.510, -1.700, 1.53, 4.52, 0.393, -0.65, -0.6, 0.1)^\top, \quad (26)$$

$$\delta \vec{b} = (0.013, 0.054, 0.13, 0.31, 0.019, 0.082, 0.19, 0.83, 0.017, 0.23, 1.5, 2.8)^\top. \quad (27)$$

We note that only the  $b_n^+$ 's are directly constrained by the experimental data because only  $f_+(q^2)$  contributes to  $B \rightarrow \pi \ell \bar{\nu}_\ell$  as seen in Eqs. (12), (15), (19), and (20). In addition, the  $b_n^0$ 's are indirectly constrained through the relation  $f_0(0) = f_+(0)$ . The tensor form factor  $f_T(q^2)$  is determined thoroughly by the lattice simulation and this explains the relatively large errors of the  $b_n^T$ 's.

The covariance matrix is given by  $V_{ij} = \rho_{ij} \delta b_i \delta b_j$  with

$$\rho_{\text{lat.+exp.}} = \begin{pmatrix} \rho_{+,0} & \mathbf{0}_{8 \times 4} \\ \mathbf{0}_{4 \times 8} & \rho_T \end{pmatrix}, \quad (28)$$

$$\rho_{+,0} = \begin{pmatrix} 1 & 0.14 & -0.455 & -0.342 & 0.224 & 0.174 & 0.047 & -0.033 \\ & 1 & -0.789 & -0.874 & -0.068 & 0.142 & 0.025 & -0.007 \\ & & 1 & 0.879 & -0.051 & -0.253 & 0.098 & 0.234 \\ & & & 1 & 0.076 & 0.038 & 0.018 & -0.2 \\ & & & & 1 & -0.043 & -0.604 & -0.388 \\ & & & & & 1 & -0.408 & -0.758 \\ & & & & & & 1 & 0.457 \\ & & & & & & & 1 \end{pmatrix}, \quad (29)$$

$$\rho_T = \begin{pmatrix} 1 & 0.4 & 0.204 & 0.166 \\ & 1 & 0.862 & 0.806 \\ & & 1 & 0.989 \\ & & & 1 \end{pmatrix}, \quad (30)$$

where the  $\rho$ 's are symmetric correlation matrices. Here, we have omitted the correlations between the  $+, 0$  sector and the  $T$  sector, because the covariance matrix turns out not to be positive semidefinite if all the correlations reported in Refs. [11,21] are taken. Negative eigenvalues of a covariance matrix may arise due to the fluctuation of eigenvalues. In such a case, the correlation is less significant and could be neglected.

The error of  $\vec{b}$  induces the uncertainty in both the SM and new physics contributions in the observable  $R_\pi$ . To estimate the uncertainty of  $R_\pi$ , we calculate its variance  $V(R_\pi)$  assuming the Gaussian distribution:

$$V(R_\pi) = \int d\vec{b} \left( R_\pi(\vec{b}) - R_\pi(\vec{b}_0) \right)^2 \exp \left[ -\frac{1}{2} \chi^2(\vec{b}) \right], \quad (31)$$

$$\chi^2(\vec{b}) = \left( \vec{b} - \vec{b}_0 \right)^\top V(\vec{b})^{-1} \left( \vec{b} - \vec{b}_0 \right). \quad (32)$$

The theoretical uncertainty of  $R_\pi$  is thus given by  $\delta R_\pi = (V(R_\pi))^{1/2}$ .

### 2.3. $B^- \rightarrow \tau^- \bar{\nu}_\tau$

The branching fraction of  $B^- \rightarrow \tau^- \bar{\nu}_\tau$  in the effective Lagrangian in Eq. (3) is expressed as

$$\mathcal{B}(B \rightarrow \tau \bar{\nu}_\tau) = \frac{\tau_{B^-} G_F^2 |V_{ub}|^2 f_B^2}{8\pi} m_B m_\tau^2 \left( 1 - \frac{m_\tau^2}{m_B^2} \right)^2 |1 + r_{\text{NP}}|^2, \quad (33)$$

where  $\tau_{B^-}$  is the charged  $B$  meson lifetime,  $f_B$  is the  $B$  meson decay constant, and  $r_{\text{NP}}$  represents the new physics effect,

$$r_{\text{NP}} = C_{V_1} - C_{V_2} + \frac{m_B^2}{m_b m_\tau} (C_{S_1} - C_{S_2}). \quad (34)$$

We note that the tensor operator  $\mathcal{O}_T$  does not contribute to this decay mode.

The dominant sources of theoretical uncertainty in  $\mathcal{B}(B \rightarrow \tau \bar{\nu}_\tau)$  are  $f_B$  and  $|V_{ub}|$ . The FLAG working group gives an average of lattice QCD results [23–27] as  $f_B = (192.0 \pm 4.3) \text{ MeV}$  [28], which is consistent with another average [29]. As for  $|V_{ub}|$ , the tension among the values determined from  $B \rightarrow \pi \ell \bar{\nu}_\ell$  (exclusive),  $B \rightarrow X_u \ell \bar{\nu}_\ell$  (inclusive), and the fit of the unitarity triangle is still unsolved. To avoid the uncertainty due to  $|V_{ub}|$ , the following ratio of pure- and semi-leptonic decay rates is defined as [30]

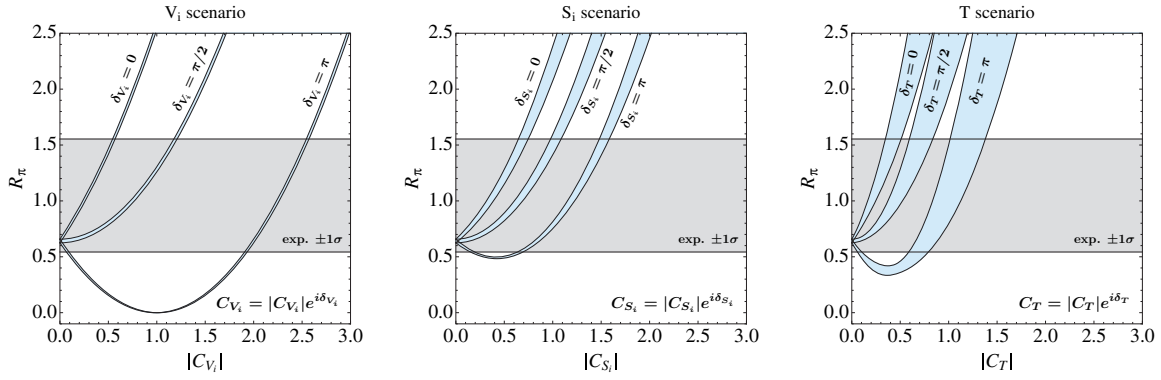
$$R_{\text{ps}} = \frac{\Gamma(B^- \rightarrow \tau^- \bar{\nu}_\tau)}{\Gamma(\bar{B}^0 \rightarrow \pi^+ \ell^- \bar{\nu}_\ell)} = \frac{\tau_{B^0}}{\tau_{B^-}} \frac{\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B}^0 \rightarrow \pi^+ \ell^- \bar{\nu}_\ell)}. \quad (35)$$

The remaining sources of theoretical uncertainty in  $R_{\text{ps}}$  are  $f_B$  and the form factor  $f_+(q^2)$  involved in the denominator. For the latter, we use the lattice result described above.

## 3. Numerical results

### 3.1. New physics scenarios

We consider new physics scenarios such that only one of the operators  $\mathcal{O}_X$  ( $X = V_1, V_2, S_1, S_2, T$ ) is dominant in the new physics sector. These scenarios are constrained by both  $B \rightarrow \pi \tau \bar{\nu}_\tau$  and  $B \rightarrow \tau \bar{\nu}_\tau$ , except for the tensor operator scenario, in which  $B \rightarrow \tau \bar{\nu}_\tau$  is not altered.



**Fig. 1.** New physics effects on  $R_\pi$  in the  $V_i$ ,  $S_i$ , and  $T$  scenarios. Three values of the complex phase,  $\delta_X = 0, \pi/2$ , and  $\pi$ , are chosen. The blue regions represent the theoretical predictions on  $R_\pi$  taking the theoretical uncertainty ( $\pm 1\sigma$ ) into account. The gray regions show the current experimental bound,  $R_\pi^{\text{exp.}} \simeq 1.05 \pm 0.51$ .

First, we present numerical formulae for the theoretical uncertainties  $\delta R_\pi$  obtained by computing the variance in Eq. (31) for each scenario:

$$\delta R_\pi(C_{V_i}, C_{X \neq V_i} = 0) \simeq \delta R_\pi^{\text{SM}} |1 + C_{V_i}|^2, \quad (36)$$

$$\begin{aligned} \delta R_\pi(C_{S_i}, C_{X \neq S_i} = 0) \simeq \delta R_\pi^{\text{SM}} \left( 1 + 7(\text{Re } C_{S_i}) + 15(\text{Re } C_{S_i})^2 + 9|C_{S_i}|^2 \right. \\ \left. + 35(\text{Re } C_{S_i})|C_{S_i}|^2 + 21|C_{S_i}|^4 \right)^{1/2}, \end{aligned} \quad (37)$$

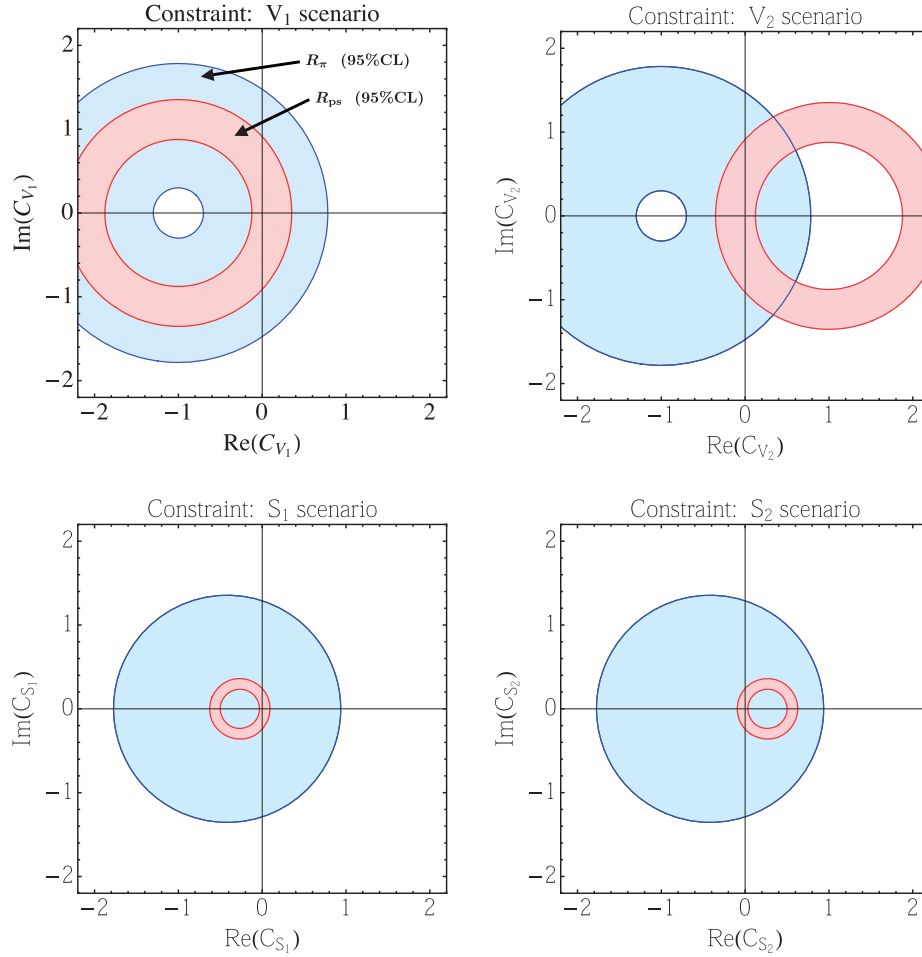
$$\begin{aligned} \delta R_\pi(C_T, C_{X \neq T} = 0) \simeq \delta R_\pi^{\text{SM}} \left( 1 + 4(\text{Re } C_T) + 350(\text{Re } C_T)^2 + 11|C_T|^2 \right. \\ \left. + 1372(\text{Re } C_T)|C_T|^2 + 1484|C_T|^4 \right)^{1/2}, \end{aligned} \quad (38)$$

where  $\delta R_\pi^{\text{SM}} \simeq 0.016$  represents the uncertainty in the SM, which is consistent with the value in Refs. [10, 16]. We observe that the contribution of the tensor operator is rather uncertain because of the less-determined form factor  $f_T(q^2)$  as mentioned above.

In Fig. 1, we show  $R_\pi$  in our new physics scenarios as functions of  $|C_X|$  for three representative values of the complex phase (defined by  $C_X = |C_X|e^{i\delta_X}$ ) as indicated. The light blue regions are the theoretical predictions with the  $\pm 1\sigma$  uncertainties evaluated with Eqs. (36)–(38). The gray region expresses the present experimental bound at the  $1\sigma$  level as is estimated in Sect. 1. One finds that the theoretical uncertainty in the vector scenarios is fairly small compared with the experimental error, whereas that in the tensor scenario is significant.<sup>5</sup>

One may observe in Eqs. (12), (13), and (34) that  $\mathcal{O}_{V_1}$  and  $\mathcal{O}_{V_2}$  have the same contribution to  $B \rightarrow \pi \tau \bar{\nu}_\tau$  whereas their contributions to  $B \rightarrow \tau \bar{\nu}_\tau$  possess opposite signs to each other. This is simply because the (axial-)vector current  $\bar{u}\gamma^\mu b$  ( $\bar{u}\gamma^\mu \gamma^5 b$ ) contributes only to the  $B \rightarrow \pi$  transition ( $B$  annihilation). Thus, the vector and the axial-vector parts of the new physics, namely  $C_V = (C_{V_1} + C_{V_2})/2$  and  $C_A = (C_{V_2} - C_{V_1})/2$  are separately constrained by  $B \rightarrow \pi \tau \bar{\nu}_\tau$  and  $B \rightarrow \tau \bar{\nu}_\tau$  respectively. The same argument applies to  $\mathcal{O}_{S_1}$  and  $\mathcal{O}_{S_2}$ . The (pseudo)scalar part,

<sup>5</sup> The uncertainty in the bottom quark mass, which is fixed in the present work, increases the theoretical uncertainties in the scalar scenarios. Varying  $m_b$  by  $\pm 200$  MeV changes at most  $\mathcal{B}(B \rightarrow \pi \tau \bar{\nu}_\tau)$  and  $\mathcal{B}(B \rightarrow \tau \bar{\nu}_\tau)$  by  $\pm 6\%$  and  $\pm 7\%$  respectively for  $|C_{S_i}| < 1$ .



**Fig. 2.** Allowed regions from  $R_\pi$  and  $R_{ps}$  for  $V_1$ ,  $V_2$ ,  $S_1$ , and  $S_2$  scenarios. The light blue region is allowed from the value of  $R_\pi$  derived from the Belle experiment at 95% CL, where the theoretical uncertainty is also taken for the evaluation. The light red region is consistent with the experimental value of  $R_{ps}$  taking into account the theoretical uncertainty described in the main text.

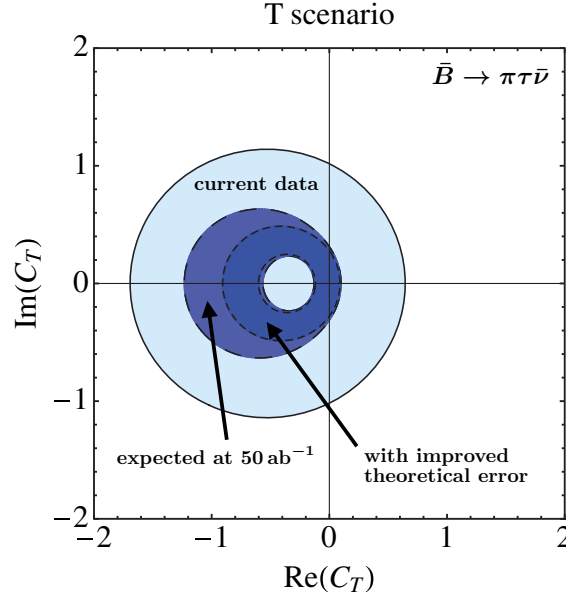
$C_S = (C_{S_1} + C_{S_2})/2$  ( $C_P = (C_{S_1} - C_{S_2})/2$ ) is constrained by  $B \rightarrow \pi \tau \bar{\nu}_\tau$  ( $B \rightarrow \tau \bar{\nu}_\tau$ ). As stressed above, the tensor operator  $\mathcal{O}_T$  contributes only to  $B \rightarrow \pi \tau \bar{\nu}_\tau$ .

### 3.2. Present constraints

The current experimental result for  $R_\pi$  is given in Sect. 1,  $R_\pi^{\text{exp.}} \simeq 1.05 \pm 0.51$ . As for  $R_{ps}$ , we obtain  $R_{ps}^{\text{exp.}} = 0.73 \pm 0.14$ , while the SM prediction is  $R_{ps}^{\text{SM}} = 0.574 \pm 0.046$  including the uncertainties of  $f_B$  and  $f_+(q^2)$ . Given these experimental data, we present constraints on the Wilson coefficients  $C_X$  for  $V_1$ ,  $V_2$ ,  $S_1$ , and  $S_2$  scenarios in Fig. 2, in which the 95% CL allowed regions by  $R_\pi$  and  $R_{ps}$  for each scenario are shown. The light blue and red regions are allowed by  $R_\pi$  and  $R_{ps}$ , respectively, taking both the theoretical and experimental uncertainties into account.

The current data of  $R_\pi$  excludes part of the region of  $|C_X| \sim O(1)$ , which is roughly the same order of magnitude as the SM contribution. The excluded region by  $R_\pi$  does not exceed the one by  $R_{ps}$  in the  $V_1$  scenario, but their difference is not so significant. As for the  $V_2$  scenario,  $R_\pi$  and  $R_{ps}$  are complementary because the signs of the new physics contributions relative to the SM ones are opposite in these observables as seen in Eqs. (12), (13), and (34). The  $S_1$  and  $S_2$  scenarios are





**Fig. 3.** Allowed regions of the tensor scenario for the recent experimental data of  $R_\pi$  and expected improvements in future. The present allowed region at the 95% CL is depicted in light blue. The (dark) blue region enclosed by the dashed (dotted) curves shows the allowed region expected at SuperKEKB/Belle II with  $50 \text{ ab}^{-1}$  data (and a theoretical uncertainty reduced by a factor of 2).

constrained more tightly by  $R_{\text{ps}}$  because of the chiral enhancement of the pseudoscalar contribution in the purely leptonic decay.

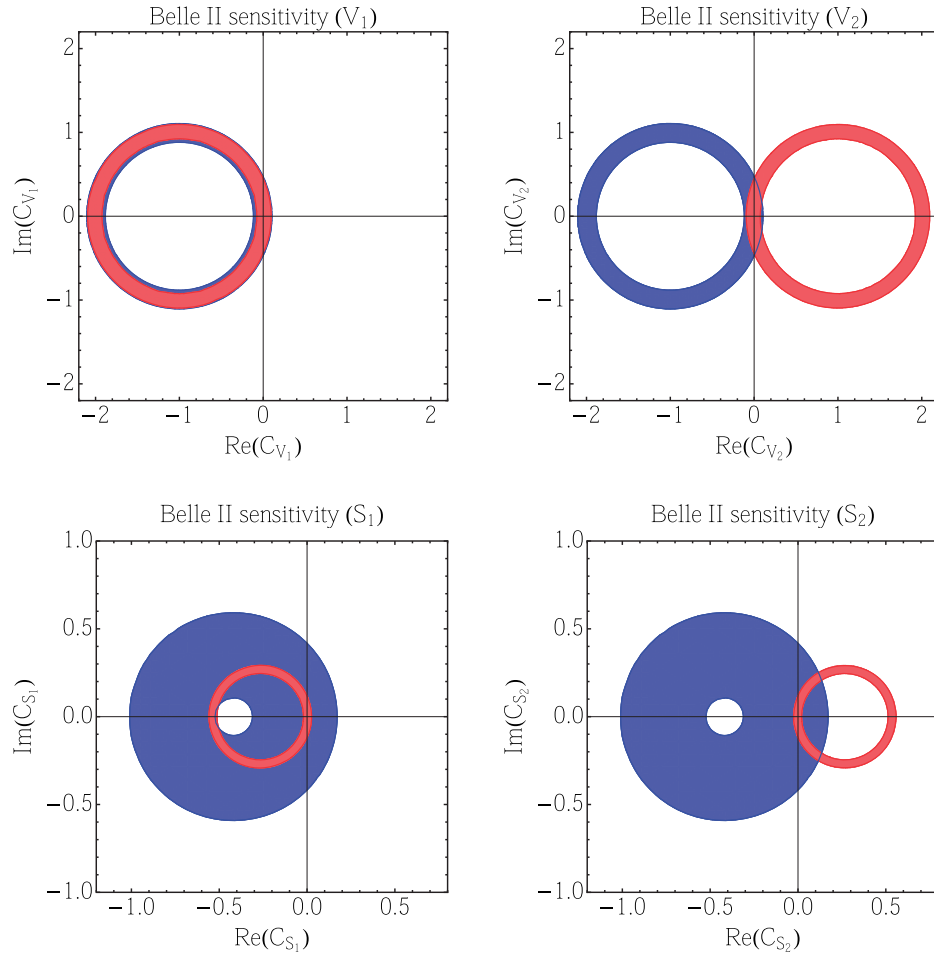
In Fig. 3, we show the  $R_\pi$  constraint on  $C_T$ . The light blue region represents the 95% CL allowed region. (The darker blue regions will be explained below.) We see that the present constraint is nontrivial and comparable to the other scenarios even though the theoretical uncertainties are considerably larger. This is because the tensor operator (as normalized in Eq. (3)) tends to give a larger contribution to  $R_\pi$ . The tensor contribution is expected to be more significant in the  $B \rightarrow V$  transitions, such as  $\bar{B} \rightarrow \rho$  as in the case of  $\bar{B} \rightarrow D^* \tau \bar{\nu}_\tau$ .

### 3.3. Future prospect

From now on, we discuss expected data of the relevant observables at SuperKEKB/Belle II and estimate the possible sensitivity to the new physics scenarios. The current experimental value of  $R_\pi$  given in Eq. (1) is obtained with  $\sim 1 \text{ ab}^{-1}$  data. We expect  $\sim 50 \text{ ab}^{-1}$  at the SuperKEKB/Belle II experiment. To evaluate the expected sensitivity of  $R_\pi$  to the new physics scenarios at SuperKEKB/Belle II, we assume that both the statistical and systematic errors in the experiment are reduced with increasing luminosity as  $1/\sqrt{\mathcal{L}}$  and that the central value coincides with the SM prediction. Namely, we employ  $R_\pi^{\text{Belle II}} = 0.641 \pm 0.071$ . Applying a similar argument to  $B \rightarrow \tau \nu$  and  $B \rightarrow \pi \ell \nu$  gives  $R_{\text{ps}}^{\text{Belle II}} = 0.574 \pm 0.020$ .<sup>6</sup>

The 95% CL expected constraints on the Wilson coefficients with these “future” experimental data are shown in Fig. 4 for the  $V_1$ ,  $V_2$ ,  $S_1$ , and  $S_2$  scenarios. A new physics contribution beyond the blue

<sup>6</sup> The expected Belle II sensitivity for  $\mathcal{B}(B^- \rightarrow \tau^- \bar{\nu})$  has been recently studied in Ref. [31] with Monte Carlo simulation assuming  $0.5 \text{ ab}^{-1}$ . Our estimation and their result of the experimental uncertainty scaled to  $50 \text{ ab}^{-1}$  are consistent.



**Fig. 4.** Sensitivity to the new physics scenarios in terms of the 95% CL allowed range of  $C_X$  expected at the SuperKEKB/Belle II with  $50 \text{ ab}^{-1}$  of accumulated data. The “future” experimental data are given as explained in the main text. A new physics contribution for the outside regions of the blue and red colors can be probed by  $R_\pi$  and  $R_{\text{ps}}$ , respectively, at  $50 \text{ ab}^{-1}$  of Belle II.

and red regions can be probed by measuring  $R_\pi$  and  $R_{\text{ps}}$ , respectively at Belle II. Each allowed region is annulus-like in the complex plane of  $C_X$ . For the  $V_1$  scenario, the new physics sensitivities of  $R_\pi$  and  $R_{\text{ps}}$  are almost degenerate and the region around  $C_{V_1} \sim -2$  of large negative interference with the SM contribution is allowed by both of them. On the other hand, in the  $V_2$  scenario, such regions of  $C_{V_2} \sim -2$  for  $R_\pi$  and  $C_{V_2} \sim +2$  for  $R_{\text{ps}}$  are incompatible with each other, as is already seen in the current constraint shown in Fig. 2. For the scalar scenarios, the regions of  $C_{S_1} = C_{S_2} \sim -0.8$  in  $R_\pi$  and  $C_{S_1} = -C_{S_2} \sim -0.5$  in  $R_{\text{ps}}$  are of large negative interference. As is seen in the figures, we can test such a region for the  $S_2$  scenario by combining  $R_\pi$  and  $R_{\text{ps}}$  while the sensitivity is relatively weak in the  $S_1$  scenario. Therefore, the constraints from  $R_\pi$  and  $R_{\text{ps}}$  are complimentary and measuring both of them at SuperKEKB/Belle II is meaningful to reduce allowed parameter regions, in particular for the  $V_2$  and  $S_2$  scenarios.

As for the tensor scenario, we also show the expected allowed region for  $C_T$  in Fig. 3. The blue region with the dashed boundary indicates the one that can be tested with  $50 \text{ ab}^{-1}$ , and the darker blue region with the dotted curve corresponds to the result for the case that the theoretical uncertainty in Eq. (38) is reduced by a factor of 2. As is explained in Sect. 3.1, the tensor scenario suffers from

**Table 1.** Sensitivity to the new physics scenarios in terms of the 95% CL allowed range of  $C_X$  expected at the SuperKEKB/Belle II with  $50 \text{ ab}^{-1}$  of accumulated data; the “future” experimental data are given as explained in the main text and the coefficient  $C_X$  is assumed to be real.

NP scenario	$R_\pi^{\text{Belle II}} = 0.641 \pm 0.071$ and $R_{\text{ps}}^{\text{Belle II}} = 0.574 \pm 0.020$	$R_{\text{pl}}^{\text{Belle II}} = 222 \pm 47$
$C_{V_1}$	$[-0.08, 0.09]; [-2.09, -1.92]$	$[-0.23, 0.19]; [-2.19, -1.77]$
$C_{V_2}$	$[-0.09, 0.08]$	$[-0.19, 0.23]; [1.77, 2.19]$
$C_{S_1}$	$[-0.03, 0.03]; [-0.55, -0.52]$	$[-0.06, 0.05]; [-0.58, -0.47]$
$C_{S_2}$	$[-0.03, 0.03]$	$[-0.05, 0.06]; [0.47, 0.58]$
$C_T$	$[-0.13, 0.10]; [-1.23, -0.56]$	—

the larger theoretical uncertainty in  $R_\pi$  so that we can see the significant effect of the reduction of the theoretical uncertainty. We also find that the present theoretical uncertainties for the vector and scalar scenarios are sufficiently smaller than the future (expected) experimental uncertainties.<sup>7</sup> We note that another observable such as  $\mathcal{B}(B \rightarrow \rho \tau \bar{\nu})$  is necessary to exclude the region of large negative interference of  $C_T \sim -0.7$ .

In Table 1, we present the combined limits of the allowed ranges for  $C_X$  (taken real) in order to quantify the expected sensitivities at SuperKEKB/Belle II. It turns out that, focusing on the vicinity of the origin, the region of  $|C_X| \gtrsim 0.03$  can be probed in the scalar scenarios. As for the vector and tensor scenarios, the Belle II sensitivity is  $|C_X| \sim 0.1$ .

The muonic mode  $B \rightarrow \mu \bar{\nu}_\mu$  may also play an important role at SuperKEKB/Belle II. At present, this process has not yet been observed and the current upper limit on the branching ratio is reported as  $\mathcal{B}(B \rightarrow \mu \bar{\nu}_\mu)^{\text{exp.}} < 1 \times 10^{-6}$  at 90% CL [32–34]. This result may be compared with the SM prediction  $\mathcal{B}(B \rightarrow \mu \bar{\nu}_\mu)^{\text{SM}} = (0.41 \pm 0.05) \times 10^{-6}$  and thus, we expect that  $B \rightarrow \mu \bar{\nu}_\mu$  will be observed with a meaningful statistical significance at SuperKEKB/Belle II. Accordingly, we introduce the pure-leptonic ratio

$$R_{\text{pl}} = \frac{\mathcal{B}(B \rightarrow \tau \bar{\nu}_\tau)}{\mathcal{B}(B \rightarrow \mu \bar{\nu}_\mu)}, \quad (39)$$

as we defined  $R_\pi$ . In this paper, we assume contributions other than the SM do not exist in  $B \rightarrow \mu \bar{\nu}_\mu$  as well as  $B \rightarrow \pi \ell \bar{\nu}$ . From the theory side,  $R_{\text{pl}}$  is precisely evaluated as

$$R_{\text{pl}} = \frac{m_\tau^2}{m_\mu^2} \frac{(1 - m_\tau^2/m_B^2)^2}{(1 - m_\mu^2/m_B^2)^2} |1 + r_{\text{NP}}|^2 \simeq 222 |1 + r_{\text{NP}}|^2. \quad (40)$$

The dominant source of uncertainty  $f_B |V_{ub}|$  in the leptonic decay rates cancels out and hence it is free from the  $|V_{ub}|$  determinations, in which some discrepancies might still remain in the Belle II era.

Following Ref. [33], the  $1\sigma$  range of the error in  $\mathcal{B}(B \rightarrow \mu \bar{\nu}_\mu)^{\text{exp.}}$  is obtained as  $\pm 0.6 \times 10^{-6}$  at present. This is expected to be reduced as  $\pm 0.08 \times 10^{-6}$  with  $50 \text{ ab}^{-1}$  at SuperKEKB/Belle II. Applying the same procedure as with  $R_\pi$ , namely with the expected “future” data being given

<sup>7</sup> For example, the reduction of the theoretical error by factor 2, gives only 0.1% and 1% differences in the expected allowed regions for the vector and scalar scenarios, respectively.

as  $R_{\text{pl}}^{\text{Belle II}} = 222 \pm 47$ , we have evaluated the future sensitivity of the ratio  $R_{\text{pl}}$  to the new physics scenarios as shown in Table 1. One finds that the sensitivity of  $R_{\text{pl}}$  is rather ( $\sim$  factor 2) weaker than that of  $R_{\text{ps}}$ . Although  $R_{\text{ps}}$  has better performance, the ratio  $R_{\text{pl}}$  is still a good observable in the sense that it has a very accurate theoretical prediction and could be used as a consistency check.

#### 4. Summary

We have studied possible new physics in the semi- and pure-tauonic  $B$  decays,  $B \rightarrow \pi \tau \bar{\nu}_\tau$  and  $B \rightarrow \tau \bar{\nu}_\tau$ , using the model-independent effective Lagrangian including the vector ( $V_{1,2}$ ), scalar ( $S_{1,2}$ ), and tensor ( $T$ ) types of interaction. The formulae of the differential branching fractions in the presence of new physics described by the effective Lagrangian are presented with a brief summary of the hadronic form factors in the  $B \rightarrow \pi$  transition.

We have examined the ratio of the branching fraction of  $B \rightarrow \pi \tau \bar{\nu}_\tau$  to that of  $B \rightarrow \pi \ell \bar{\nu}_\ell$ ,  $R_\pi$  defined in Eq. (2), in order to reduce uncertainties in theoretical calculations in analogy with  $B \rightarrow D^{(*)} \tau \bar{\nu}_\tau$ . Using the recent results of lattice QCD studies on the relevant form factors, we have evaluated the effects of new physics in  $R_\pi$  along with its theoretical uncertainty. The theoretical uncertainties in the  $V_{1,2}$  scenarios are negligible compared to the present experimental error, and those in the  $S_{1,2}$  scenarios are sizable, but sufficiently small. In contrast, the new physics contribution in the  $T$  scenario is rather uncertain as shown in Fig. 1.

We have obtained the present constraints on the Wilson coefficients that describe possible new physics contributions,  $C_X$  ( $X = V_{1,2}, S_{1,2}, T$ ), comparing the theoretical predictions (with uncertainties mentioned above) of  $R_\pi$  and  $R_{\text{ps}}$  with the experimental data. As shown in Fig. 2, some of the regions of  $|C_X| \gtrsim O(1)$  are disfavored by the current data. The sensitivity of  $R_\pi$  in the  $V_1$  scenario is less than that of  $R_{\text{ps}}$ , but their difference is not so significant. In the  $V_2$  scenario, these two observables probe different regions of  $C_{V_2}$  and are complementary. As for the  $S_{1,2}$  scenarios,  $R_{\text{ps}}$  is more sensitive owing to the chiral enhancement. Since the tensor operator does not contribute to  $B \rightarrow \tau \bar{\nu}_\tau$ , the  $T$  scenario is constrained solely by  $R_\pi$ .

Furthermore, we have discussed the future prospects at the SuperKEKB/Belle II experiment and shown its sensitivity to new physics in terms of expected constraints on  $C_X$ . Assuming that both the statistical and systematic uncertainties in the experiment are reduced as the integrated luminosity is increased to  $50 \text{ ab}^{-1}$  and the central values are given by the SM, we have estimated the expected allowed ranges of  $C_X$  from  $R_\pi$  and  $R_{\text{ps}}$ .

It turns out that the allowed regions of  $C_X$  are significantly reduced in all the scenarios and the region of large negative interference with the SM can be excluded by combining  $R_\pi$  and  $R_{\text{ps}}$  in the  $V_2$  and  $S_2$  scenarios as shown in Figs. 3 and 4. The SuperKEKB/Belle II experiment can probe the new physics contribution of  $|C_X|$  as small as 0.03 in the scalar scenarios and  $\sim 0.1$  in the vector and tensor scenarios as seen in Table 1.

Further improvement of sensitivity may be achieved if  $R_\pi$  and  $R_{\text{pl}}$  are measured by a similar method adopted to measure  $R_{D^{(*)}}$ , namely not separate measurements of the numerator and denominator but direct measurements of the ratios. It is also desired to improve the precision of the tensor form factor as well as to evaluate the scalar form factor by lattice simulation. The latter is useful to eliminate the potential uncertainty in the bottom quark mass arising from the equation of motion. Supplemental observables such as  $\mathcal{B}(B \rightarrow \rho \tau \bar{\nu})$  and the  $q^2$  distribution of  $B \rightarrow \pi \tau \bar{\nu}$  are also helpful to further squeeze  $C_X$ , as well as to probe or exclude the region of negative interference in the  $S_1$  and  $T$  scenarios.

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